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Solution by V. M. SPUNAR, Cleveland, Ohio, and the PROPOSER.

The general term is

$$\frac{1}{(4n-3)(4n-2)(4n-1)4n} = \frac{1}{6} \left(\frac{1}{4n-3} - \frac{3}{4n-2} + \frac{3}{4n-1} - \frac{1}{4n} \right)$$

$$= \frac{1}{6} \left(\frac{2}{4n-3} - \frac{2}{4n-2} + \frac{2}{4n-1} - \frac{2}{4n} \right) - \frac{1}{6} \left(\frac{1}{4n-3} - \frac{1}{4n-1} \right) - \frac{1}{6} \left(\frac{1}{4n-2} - \frac{1}{4n} \right).$$

Therefore the series may be written

$$\frac{1}{3} \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \right) - \frac{1}{6} \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots \right) - \frac{1}{12} \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \right)$$

$$= \frac{1}{4} \log 2 - \frac{1}{6} \tan^{-1} 1 = \frac{1}{4} \log 2 - \frac{1}{24} \pi.$$

343. Proposed by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

A, on contracting to execute a piece of work for \$300 and finding after working alone one day that he had finished but 1% of the entire work, engaged B to assist him at the beginning of the second day, with the understanding, that B on each day was to do 6% as much work as had been completed previously, while A each day was to do an amount of work equal to 1% of the unfinished work at the close of the day before. At the completion of all the work the \$300 were divided between A and B in proportion to the amount of the work each had performed.

Required—(1) The number of days to do the work; (2) on which day would the daily earnings of A and B be the same; and (3) the amount of money each was paid under the agreement.

Solution by the PROPOSER.

Let $r=1\%$ and $r_1=6\%$; and let x , a variable, = the time in days to do the whole work, or 1; and let the whole work completed to the end of the days 0, 1, 2, 3, ..., x , $x+1$, ..., be represented by the functions $u_0, u_1, u_2, \dots, u_x, u_{x+1}, \dots$. The whole work completed to the end of the day $x+1$, or u_{x+1} , is equal to the work completed to the end of x days, or u_x , plus the part completed by A on the day x , or $r(1-u_x)$, plus the part completed by B on the day x , or $r_1 u_x$. Equate the functions and have:

$$u_{x+1} = u_x + r(1-u_x) + r_1 u_x \dots (1);$$

$$\text{or, } u_{x+1} - (1-r+r_1)u_x = r \dots (2).$$

Give the equation numerical values and we have

$$u_{x+1} - (1.05)u_x = 0.01 \dots (3).$$

Equation (3) belongs to the Calculus of Finite Differences. Integrate it and have

$$u_x - C(1.05)^x = -0.2 \dots (4).$$

Equation (4) is true for all values of x , and is therefore true when $x=0$. When $x=0$, $C=0.2$; and this value of C gives

$$u_x = 0.2[(1.05)^x - 1] \dots (5).$$

To find (1) the time to complete the work. When the work is completed $u_x=1$. Substitute this value of u_x in (5) and have, $(1.05)^x=6$; or $x \log(1.05) = \log 6$; or $x = \log 6 / \log(1.05) = 36.72 + \text{days}$.

To find (2), when the earnings of each are the same, in equation (1) transfer u_x to the first member and have

$$u_{x+1} - u_x = r(1 - u_x) + r_1 u_x \dots (6).$$

As u_{x+1} is the whole work completed in $x+1$ days, and u_x is the whole work completed in x days, their difference is the work completed on the day $x+1$. Substitute in the second member of (6), the value of u_x from (5) and have

$$u_{x+1} - u_x = 0.002[6 - (1.05)^x] + 0.012[(1.05)^x - 1] \dots (7).$$

In (7), for $x+1$ write x , as x is a variable, and have

$$u_x - u_{x-1} = 0.002[6 - (1.05)^{x-1}] + 0.012[(1.05)^{x-1} - 1] \dots (8).$$

The first member of (8) is the work completed in x days, and the two terms of the second member show the work completed by A and B, respectively, on the day x , and as these two terms, under the conditions of the problem, must be equal, equate them, reduce, and have

$$6 - (1.05)^{x-1} = 6[(1.05)^{x-1} - 1] \dots (9);$$

or $7(1.05)^{x-1} = 12$; or $(1.05)^{x-1} = 12 \div 7$; or $(x-1) \log(1.05) = \log(12 \div 7)$;
or $x = 1 + \log(12 \div 7) / \log(1.05) = 12.05$ days, or 12 days.

To divide the money (3) recur to equation (8) and observe that the exponents of the two terms in the second member are one degree lower than

the subscript x in u_x . This is a general law and it enables us to generate the 36 equations of work completed as x takes different values from 1 to 36.

End of 1 day, $u_1 - u_0 = 0.002[6 - (1.05)^1] + [0] \dots (10);$

End or 2 days, $u_2 - u_1 = 0.202[6 - (1.05)^1] + 0.012[(1.05)^1 - 1] \dots (11);$

.

End of 35 days, $u_{35} - u_{34} = 0.002[6 - (1.05)^{34}] + 0.012[(1.05)^{34} - 1] \dots (44);$ and

End of 36 days, $u_{36} - u_{35} = 0.002[6 - (1.05)^{35}] + 0.012[(1.05)^{35} - 1] \dots (45).$

Add the equations and have

$$u_{36} - u_0 = 0.002\{216 - [(1.05)^0 + (1.05)^1 + \dots + (1.05)^{35}]\} \\ + 0.012[(1.05)^1 + (1.05)^2 + \dots + (1.05)^{35} - 35] \dots (46).$$

Sum the first term of the second member of (46) for the work completed by A in 36 days, and sum the second term for the work completed by B in 35 days, and have:

$$\text{for A's work, } 0.002\{216 - 20[(1.05)^{36} - 1]\} = 0.2403 +; \\ \text{and for B's work, } 0.012\{21[(1.05)^{35} - 1] - 35\} = 0.7180 +.$$

A's work for 36 days + B's work for 35 days = $0.2403 + 0.7180 = 0.9583 +$.

The unfinished work = $1 - 0.9583 = 0.0417 -$; work to be finished by A and B in 0.72 day.

For A's unfinished part we have $(0.0417)(0.01)(0.72) = 0.0003$; and $0.2403 + 0.0003 = 0.2406$ = the total part completed by A.

For B's unfinished part we have $(0.8583)(0.06)(0.72) = 0.0414$; and $0.7180 + 0.0414 = 0.7594$, the total part completed by B.

A's total + B's total = $0.2406 + 0.7594 = 1$, as it should.

A's share of the money, therefore, = $\$300 \times (0.2406) = \72.18 ; and B's share = $\$300 \times (0.7594) = \227.82 .

Also solved by V. M. Spunar.

GEOMETRY.

369. Proposed by W. J. GREENSTREET, A. M., Editor, *Mathematical Gazette*, Stroud, England.

Prove by inversion that if two circles cut at a given angle, touch each a given circle, and pass each through the same fixed point, then shall the envelope of the points of contact be a conic.

No satisfactory solution of this problem has been received.